Grade 5 Mathematics, Quarter 1, Unit 1.1

Understanding Place Value of Whole Numbers and Decimals Including Rounding

Overview

Number of instructional days: 14 (1 day = 45–60 minutes)

Content to be learned

- Understand how the value of a digit in one place compares to the value of a digit to its right or left.
- Explain the difference between the value of two digits through thousandths in a multidigit number.
- Explain patterns in the number of zeros in a product when multiplying a number by powers of ten.
- Explain patterns in the placements of a decimal point when a decimal is multiplied by a power of ten.
- Explain patterns in the placements of a decimal point when a decimal is divided by a power of ten.
- Use whole number exponents to express powers of ten.

Mathematical practices to be integrated

Attend to precision.

- Use clear, precise language, explanations, definitions, and vocabulary.
- Explain the meaning of symbols.
- Calculate accurately.
- Be able to explain understanding to others.

Look for and make use of structure.

- Look for patterns to simplify.
- Break apart numbers.

Look for and express regularity in repeated reasoning.

- Look for repeated calculations/patterns.
- Accurate calculations.

Essential questions

- How could you to write a multiple of 10 as a power of 10 using exponents?
- What can I learn about patterns by looking at the placement of the decimal point after multiplying or dividing by a power of ten?
- What strategies can I use to round decimals to any place?
- How do you represent decimals in expanded notation and how is this related to place value?
- How can you use place value to round any decimal to any place value?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 5.NBT

Understand the place value system.

5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In third grade, students rounded whole numbers. Fourth-grade students generalized their understanding of place value to 1,000,000. In fourth grade, students generalized their understanding of place value for multidigit whole numbers. Students also recognized that, in a multidigit whole number, a digit in one place represents ten times what it represents in the place to its right. This supports their work with multidigit multiplication and division of decimals in fifth grade.

Current Learning

Fifth-grade students use their understanding of place value to read, write, and compare decimals to the thousandths and round decimals to any place. Fifth-grade students expand their understanding of place value as they explain the effect of multiplying or dividing by powers of ten on decimal position and the number of zeros in a product. They also use whole number exponents to denote powers of ten.

Future Learning

Students in grade 6 will fluently add, subtract, multiply, and divide multidigit decimals using the standard algorithm for each operation. Students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and negative integers in particular. This is the culminating standard for several years' worth of work relating to the domains of NBT, OA, and NF.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Students in these grades should use models and other strategies to represent and study decimal numbers. For example, they should count by tenths, (one tenths, two tenths…) verbally or use a calculator to link and relate whole numbers with decimal numbers. As students continue to count orally from nine-tenths to ten-tenths to eleven-tenths and see the calculator display change from 0.9 to 1.0 to 1.1, they see that ten-tenths is the same as one and also how it relates to 0.9 and 1.1" (p. 150).

Grade 5 Mathematics, Quarter 1, Unit 1.2

Comparing, Ordering, and Rounding Decimals

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Read decimals to thousandths place value in base-ten numeral (standard), number names or expanded form.
- Write decimals to thousandths place value in base-ten numeral (standard), number names, or expanded form.
- Compare decimals to the thousandths using <, >, and =.
- Use place value understandings to round decimals to any place.

Mathematical practices to be integrated

Attend to precision.

- Calculate accurately by rechecking for precision.
- Define mathematical symbols consistently and appropriately.

Look for and make use of structure.

- Look for patterns to simplify.
- Break numbers apart.
- Use a variety of strategies and properties to verify answers.

Look for and express regularity in repeated reasoning.

- Look for repeated calculations and patterns.
- Make accurate calculations.

Essential questions

- What do the symbols <, =, and > mean and how do you determine which symbol to use to record your comparison?
- How would you compare and order whole numbers and decimals through the thousandths?
- How are decimals and fractions alike? How are they different?
- How can decimals be rounded?
- How do I explain the meaning of a fraction and its numerator and denominator in relationship to decimals?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten **5.NBT**

Understand the place value system.

5.NBT.3 Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4 Use place value understanding to round decimals to any place.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students in fourth grade recognized that, in a multidigit whole number, a digit in one place represents ten times what it represents in the place to its right. They read and wrote multidigit whole numbers using base-ten numerals. They were able to compare two multidigit numbers based on meanings of the digits in each place using greater than, equal to, and less than symbols to record the results of comparisons. Fourth-grade students used place value understanding to round multidigit whole numbers to any place.

Current Learning

Students in grade 5 extend their understanding to read and write decimals to thousandths using base-ten numerals, number names, and expanded form. Using the symbols < , > , and = to record the results of comparisons, students compare two decimals to thousandths based on meanings of the digits in each place. Students use place value understanding to round decimals to any place.

Future Learning

Students in grade 6 will fluently add, subtract, multiply, and divide multidigit decimals using the standard algorithm for each operation. Students in grade 6 also extend the base-ten system to negative numbers. By reasoning about the standard division algorithm, students will learn in grade 7 that every fraction can be represented with a decimal that either terminates or repeats.

Additional Findings

According to *Curriculum Focal Points for Pre-K through Grade 8 Mathematics*, “They apply their understanding of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standards procedures for adding and subtracting fractions and decimals.” (p. 17)

According to *Principles and Standards for School Mathematics,* “Students should investigate the relationship between fractions and decimals, focusing on equivalence. Through a variety of activities they should understand that a fraction such as 1/2 is equivalent to 5/10 and it has a decimal representation (0.5). As they encounter a new meaning of a fraction—as a quotient of two whole numbers (1/2 equals 1 divided by 2, equals 0.5)—they can also see another way to arrive at this equivalence. By using the calculator to carry out the indicated division of familiar fractions such as 1/4, 1/3, 2/5, 1/2, and 3/4, they

can learn common fraction-decimal equivalents. They can also learn that some fractions can be expressed as terminating decimals but others cannot (p. 150).

Grade 5 Mathematics, Quarter 1, Unit 1.3

Addition and Multiplication of Whole Numbers and Decimals

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use standard algorithm to fluently multiply multidigit whole numbers.
- Connect the addition strategies to a written method.
- Connect the multiplication strategies to a written method.
- Use concrete models or drawings and strategies based on place value to add and multiply decimals to hundredths.
- Explain the reasoning used and relate the strategy to a written method when adding and multiplying decimals to hundredths.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Prove two ways.
- State justification.

Model with mathematics.

- Simplify the situation to a simpler rule allowing for revisions.
- Draw conclusions, interpret results, revise models if needed.

Attend to precision

- Label for clarification**.**
- Calculate accurately by rechecking for precision.
- Communicate formulated explanations with precision.

Essential questions

- Use standard algorithm to fluently multiply multi-digit whole numbers.
- Use concrete models or drawings and strategies based on place value to add and multiply decimals to hundredths.
- Explain the reasoning used and relate the strategy to a written method when adding and multiplying decimals to hundredths.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.7 Add, ~~subtract,~~ multiply, and ~~divide~~ decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, ~~and/or the relationship between addition and subtraction~~; relate the strategy to a written method and explain the reasoning used.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

Clarifying the Standards

Prior Learning

Students in grade 4 multiplied a whole number of up to four digits by a one-digit whole number, and multiplied two two-digit numbers, using strategies based on place value and the properties of operations. They illustrated and explained the calculation by using equations, rectangular arrays, and/or area models.

Current Learning

Students in grade 5 expand on their knowledge of adding and multiplying whole numbers to adding and multiplying decimals to hundredths, using concrete models or drawings and strategies based on place value and properties of operations. Students relate the strategy to a written method and explain the reasoning used. At this grade, expectations for decimals are limited to the thousandths; therefore, students will only multiply tenths with tenths or tenths with hundredths. They do not explore hundredths times hundredths, as the resulting product would extend beyond the thousandths place.

Before students study a general method of computing products of decimals, students should explore multiplying by 0.1 and 0.01 and be able to articulate why the product is ten or one-hundred times smaller than the multiplicand. They can then extend these understandings to multiples of 0.1 and 0.01, such as 0.2 and 0.02, 0.3 and 0.03, etc. For more insight on developing students understanding with multiplication using decimals see the PARCC document, "Progressions: K–5, Number and Operations in Base Ten," Grade 5.

Future Learning

Students in grade 6 will fluently add and multiply multidigit decimals using the standard algorithm for each operation. Students in grade 7 will extend previous understandings of multiplication and fractions to multiply rational numbers.

Additional Findings

According to *Curriculum Focal Points for Pre-K through Grade 8 Mathematics*:

"Building on their work in grade 4, student extend their understanding of place value to numbers through millions in various contexts."(p. 17)

According to *Principles and Standards for School Mathematics*:

"By the end of this grade band, students should be computing fluently with whole numbers… The computational method that the student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication…, and number relationships." (p. 152)

" Research suggests that by solving problems that require calculation, students develop methods for computing and also learn more about operations and properties. As students develop methods to solve multidigit computation problems, they should be encouraged to record and share their methods. As they do so, they can learn from one another, analyze the efficiency and generalizability of various approaches, and try one another's methods." (p. 153)

" When students leave grade 5, they should be able to solve problems involving whole number computation and should recognize that each operation will help them solve many different types of problems… With these understandings and skills, they should be able to develop strategies for computing with… decimals." (p. 149)

Grade 5 Mathematics, Quarter 1, Unit 1.4

Subtraction and Division of Whole Numbers and Decimals

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
- Illustrate and explain how to find quotients using equations.
- Relate the subtraction strategy to a written method and explain the reasoning used.
- Relate the division strategy to a written method and explain the reasoning used.
- Subtract and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Prove two ways.
- State justification.

Model with mathematics.

- Simplify the situation to a simpler rule allowing for revisions.
- Draw conclusions, interpret results, revise models if needed.

Attend to precision**.**

- Label for clarification**.**
- Calculate accurately by rechecking for precision.
- Communicate formulated explanations with precision.

Essential questions

- How can I use place value to decompose numbers to find quotients or differences?
- How can models help us understand the subtraction and division of decimals?
- How do I recognize what strategy to use for a specific problem?
- What is expanded notation and how can I use it to assist in finding the difference or quotient of multi-digit whole numbers?
- How are repeated subtraction and division related?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7 ~~Add~~, subtract, ~~multiply~~, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Clarifying the Standards

Prior Learning

Fourth grade students found whole number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Fourth-grade students illustrated and explained the calculations by using equations, rectangular arrays, and/or area models.

Current Learning

In grade 5, students find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Students in grade 5 illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. Students in grade 5 subtract and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; they also relate the strategy to a written method and explain the reasoning used.

Division strategies in grade 5 entail decomposing the dividend into like base-ten units and applying the distributive property. Additionally, division can also be thought of as finding the missing factor in a related multiplication fact.

Future Learning

In grade 6, students will fluently divide multidigit numbers using the standard algorithm. Students will fluently subtract and divide multidigit decimals using the standard algorithm for each operation.

Additional Findings

According to *Curriculum Focal Points for Pre-K through Grade 8 Mathematics*:

"Building on their work in grade 4, student extend their understanding of place value to numbers through millions in various contexts." (p. 17)

According to *Principles and Standards for Teaching Mathematics*:

"By the end of this grade band, students should be computing fluently with whole numbers… The computational method that the student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships." (p. 152)

"Research suggests that by solving problems that require calculation, students develop methods for computing and also learn more about operations and properties. As students develop methods to solve multidigit computation problems, they should be encouraged to record and share their methods. As they do so, they can learn from one another, analyze the efficiency and generalizability of various approaches, and try one another's methods." (p. 153)

"When students leave grade 5, they should be able to solve problems involving whole number computation and should recognize that each operation will help them solve many different types of problems… With these understandings and skills, they should be able to develop strategies for computing with… decimals." (p. 149)